

# Constraints on the NMSSM from the oblique parameters

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**ABSTRACT:** Electroweak precision measurements, encoded in the oblique parameters, give strong constraints on physics beyond the Standard Model. The oblique parameters  $S$ ,  $T$ ,  $U$  ( $V$ ,  $W$ ,  $X$ ) are calculated in the next-to-minimal supersymmetric model (NMSSM). We outline the calculation of the oblique parameters in terms of one-loop gauge-boson selfenergies and find sensitive restrictions for the NMSSM parameter space.

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## 1 Introduction

The precision measurements of the electroweak parameters give stringent constraints on physics beyond the Standard Model (SM). A very elegant method to systematically confront the electroweak precision measurements with new physics is given by the oblique parameters  $S$ ,  $T$ ,  $U$  [1–3]. These three parameters allow to restrict any physics beyond the SM, under the following three conditions:

- The physics beyond the SM has to obey  $SU(2)_L \otimes U(1)_Y$  gauge symmetry, that is, there are no additional electroweak gauge bosons compared to the SM.
- The couplings of new particles to light fermions have to be suppressed. That is, the main contribution of couplings beyond the SM to four-fermion scattering originates from the change in the self-energies of the gauge-boson propagators. These contributions are called *oblique* corrections. The suppressed contributions which may for instance appear in box diagrams with four external fermions or in vertex corrections are called *non-oblique* corrections.
- New physics enters only at a scale large compared to the electroweak scale.

From the second condition it is clear that the oblique parameters are expressed in terms of gauge-boson self-energies, as was shown in detail in Ref. [3]. The main argument is that the electroweak precision measurements probe weak-interaction processes with light external fermions of mass  $m_f$  (at cms energies on the electroweak scale), wherein vertex- and box-type correction are suppressed by factors of  $m_f^2/m_Z^2$  as compared to the self-energy loop corrections.

However, many models beyond the SM are expected to have effects at a scale not too far from the electroweak scale, which is given by the vacuum-expectation value of the neutral SM

Higgs boson component  $v_0 \approx 174$  GeV. In order to weaken the third condition for the oblique parameters  $S, T, U$  above, allowing new physics to enter already at a scale somewhat larger than the electroweak scale, the oblique parameters were extended to the six parameters  $S, T, U, V, W, X$  [4, 5]. The explicit expressions for these oblique parameters read

$$\begin{aligned}
S &= \frac{4s_W^2 c_W^2}{\alpha} \left[ \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} - \frac{c_W^2 - s_W^2}{s_W c_W} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right], \\
T &= \frac{1}{\alpha} \left[ \frac{\Pi_{WW}(0)}{m_W^2} - \frac{\Pi_{ZZ}(0)}{m_Z^2} \right], \\
U &= \frac{4s_W^2}{\alpha} \left[ \frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2} - c_W^2 \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} - 2s_W c_W \Pi'_{Z\gamma}(0) - s_W^2 \Pi'_{\gamma\gamma}(0) \right], \\
V &= \frac{1}{\alpha} \left[ \Pi'_{ZZ}(m_Z^2) - \frac{\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2} \right], \\
W &= \frac{1}{\alpha} \left[ \Pi'_{WW}(m_W^2) - \frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2} \right], \\
X &= -\frac{s_W c_W}{\alpha} \left[ \frac{\Pi_{Z\gamma}(m_Z^2)}{m_Z^2} - \Pi'_{Z\gamma}(0) \right].
\end{aligned} \tag{1.1}$$

The quantities  $\Pi_{G_1 G_2}(s)$  with  $G_{1/2} \in \{\gamma, W, Z\}$  denote the new contributions to the transverse part of the self-energies at a momentum-squared scale  $s$  – compared to the SM,

$$\Pi_{G_1 G_2}(s) = \Pi_{G_1 G_2}^{\text{new}}(s) - \Pi_{G_1 G_2}^{\text{SM}}(s). \tag{1.2}$$

The derivatives of the self-energies  $\Pi_{G_1 G_2}(s)$  with respect to the scale  $s$  are denoted by  $\Pi'_{G_1 G_2}(s_0) = d\Pi_{G_1 G_2}(s)/ds|_{s=s_0}$ . The fact that only relatively few parameters (besides  $\Pi(s)$  for  $s \in \{0, m_W^2, m_Z^2\}$  only  $\Pi'(s)$  at the same low-energy scales) enter in Eq. (1.1) reflects the observation that precision measurements are made only by two-particle scatterings on light fermions at those few scales, as explained in detail in Ref. [4]. Finally,  $s_W = \sin(\theta_W)$  and  $c_W = \cos(\theta_W)$  contain the usual weak Weinberg mixing angle  $\theta_W$ , and  $\alpha$  denotes the fine-structure constant.

Having defined the oblique parameters, electroweak precision observables, like for instance the  $W^\pm$ -boson mass, may be expressed in terms of these parameters. Constraints on the oblique parameters are gained via a global fit to the electroweak precision measurements; see e.g. Ref. [6]. Being exactly zero within the SM, these global fits result in error bands for the six parameters of Eqs. (1.1), see Eq. (3.1) below, hence potentially constraining the size of effects from new physics.

In this paper we compute the oblique parameters  $S, T, U, V, W, X$  of Eq. (1.1) in the next-to minimal supersymmetric extension of the SM (NMSSM); for reviews of the NMSSM, we refer to Refs. [7, 8]. The NMSSM receives a lot of attention in recently – in particular, it possesses a scale invariant superpotential, a much richer Higgs sector, and a fifth neutralino compared to the minimal supersymmetric extension (MSSM). Noting that in the fermion–fermion interactions there appear in principle also non-oblique corrections in the NMSSM, here we assume that the non-oblique corrections are negligible.

set#	$M_0^{\text{GUT}}$	$M_{1/2}^{\text{GUT}}$	$A_0^{\text{GUT}}$	$A_\kappa^{\text{GUT}}$	$\tan(\beta)^{\text{MSUSY}}$	$\text{sgn}(\mu)$	$\lambda^{\text{MSUSY}}$
1	500	500	-800	-100	5	+	0.15
2	500	500	-800	-1500	1.7	+	0.5
3	100	200	-700	-75	5	+	0.2

**Table 1.** Parameter values for the example studies in case of the constrained NMSSM ( $M_i$  and  $A_i$  in GeV). These sets are inspired by the ranges given in Figs. 1–3 of Ref. [17].

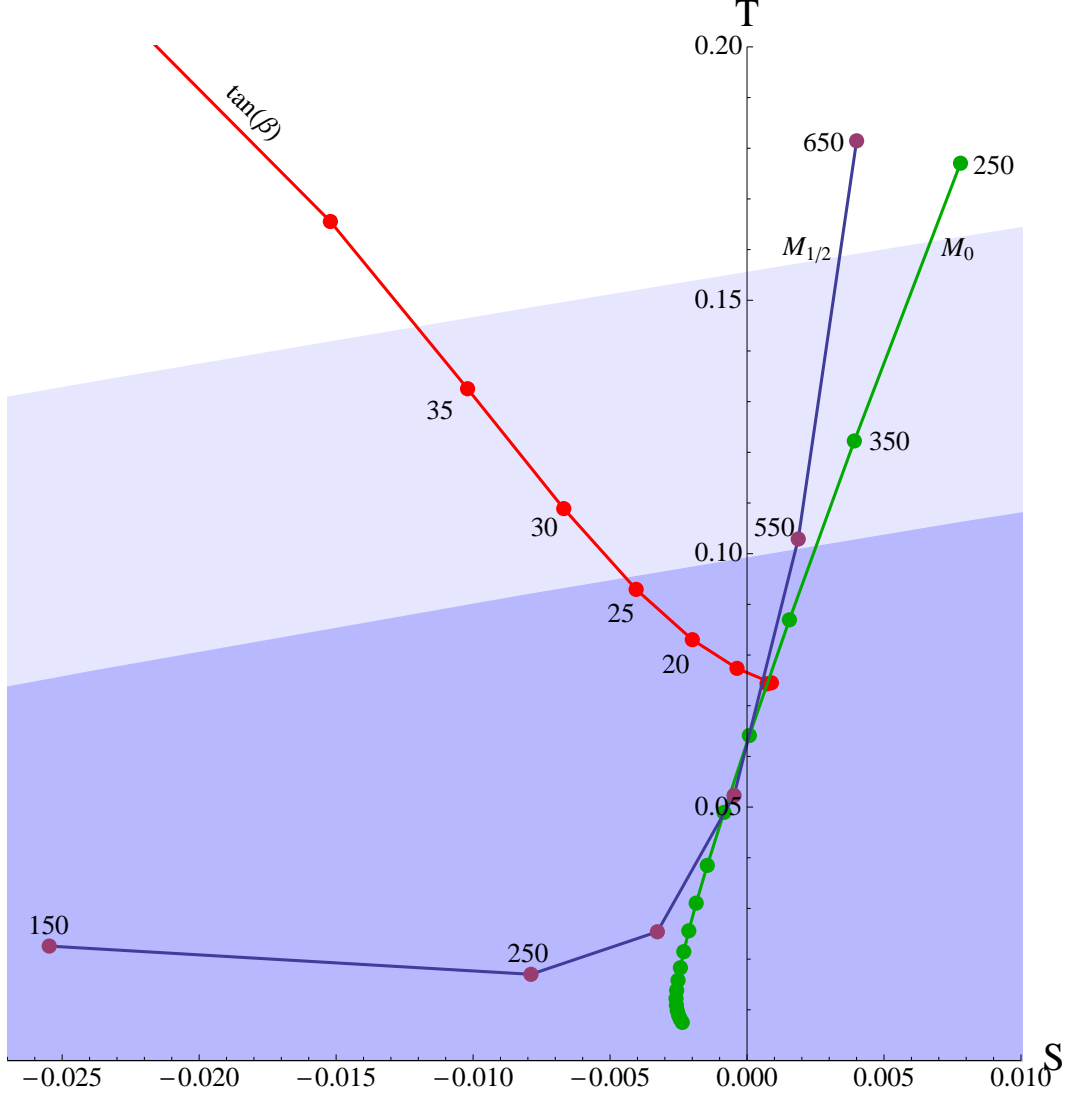
There are several computations of electroweak precision observables in the NMSSM. Let us mention the study of the  $Z^0$  boson width [9] as well as the study of the  $W^\pm$ -mass and the  $Z$  boson decay into leptons [10]. Let us remark that there exist similar approaches for the case of the MSSM [11–14]. Since the parameter space of the NMSSM is very large, there are different approaches to phenomenological studies of this model. In Refs. [15–17], for instance, the constrained version of the NMSSM is considered where it is assumed that various masses and couplings unify at the GUT scale. Another approach is to consider specific benchmarks scenarios, representing different regimes in parameter space [18, 19]. In our numerical examples below, we shall adopt the former approach.

## 2 Details of the calculation

For the prediction of the oblique parameters of Eq. (1.1) we need to compute the transverse parts of the one-loop self-energies  $\Pi_{G_1 G_2}(s) = \Pi_{G_1 G_2}^{\text{NMSSM}}(s) - \Pi_{G_1 G_2}^{\text{SM}}(s)$ , where  $G_1, G_2$  denote the gauge bosons  $\gamma, W^\pm, Z^0$ . The self-energies with exclusively leptons, quarks, and gauge bosons in the loops are exactly the same in  $\Pi_{G_1 G_2}^{\text{NMSSM}}(s)$  and  $\Pi_{G_1 G_2}^{\text{SM}}(s)$  and therefore do not need to be evaluated. As a consistency check, however, we confirmed this analytically.

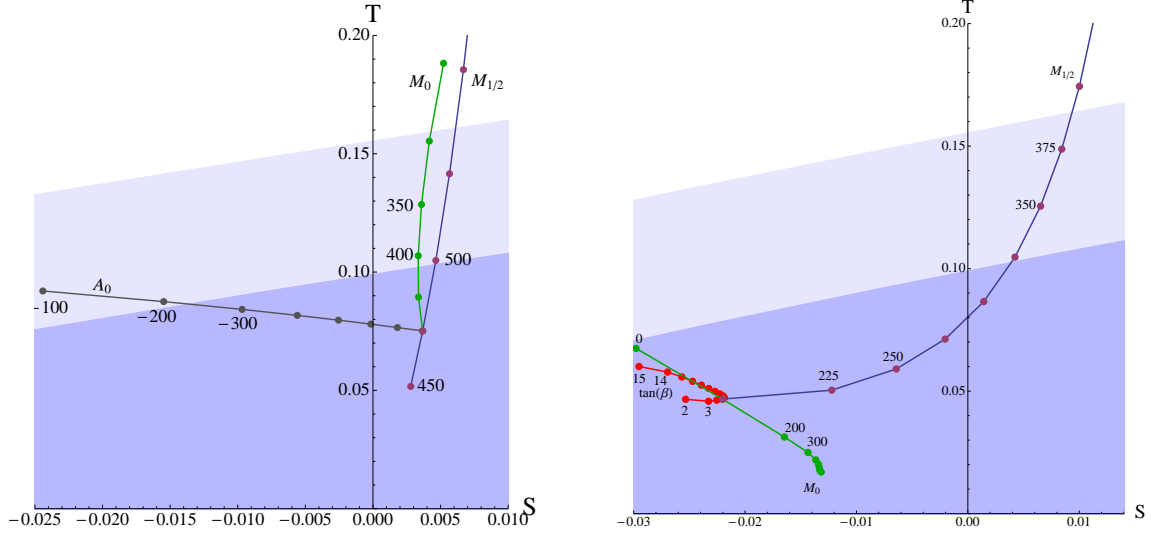
Since the Higgs sector of the NMSSM is not a simple extension of the SM Higgs sector, we have to consider in  $\Pi_{G_1 G_2}^{\text{SM}}(s)$  all contributions which contain the SM Higgs boson  $H_{\text{SM}}$ . In the self-energies of the NMSSM we have to consider all contributions which involve scalar neutrinos  $\tilde{\nu}$ , scalar leptons  $\tilde{l}$ , scalar up- and down- type quarks  $\tilde{u}, \tilde{d}$ , neutralinos  $\chi^0$ , charginos  $\chi^\pm$ , the neutral Higgs bosons  $H_1, H_2, H_3, A_1, A_2$ , the pair of charged Higgs bosons  $H^\pm$  and the Goldstone bosons  $G^0, G^\pm$ . All Feynman diagrams of the self-energy contributions to the oblique parameters are shown in App. A. Let us note that we consider the most general NMSSM in our computation. In particular we allow for CP violation in the Higgs sector, such that the neutral Higgs bosons  $H_i/A_j$  are not necessarily CP even/odd, respectively; for details see for instance Ref. [7].

The NMSSM Feynman rules are implemented in the **FeynRules** program package [20, 21] following the conventions of Ref. [22]. As a caveat, let us remark here that the Goldstone components of the neutral Higgs boson squared mixing matrix have to be carefully constructed such as to guarantee unitarity, which is violated by the parameters chosen in the model file `nmssm.fr`. We link this list of Feynman rules with the packages **FeynArts/FormCalc** [23],



**Figure 1.** Oblique parameters  $S$  and  $T$  in the NMSSM in the constrained case with parameters  $\tan(\beta) = 5$ ,  $M_0 = M_{1/2} = 500$  GeV,  $A_0 = -800$  GeV,  $A_\kappa = -100$  GeV,  $\text{sgn}(\mu) = +$ , and  $\lambda = 0.15$  (first row of Tab. 1) with the parameters  $\tan(\beta)$ ,  $M_0$ ,  $M_{1/2}$  varied as indicated in the figure. The shaded regions show the  $1\sigma$  and  $2\sigma$  error ellipses of the electroweak precision measurements fitted to  $S$  and  $T$  corresponding to Eq. (3.1) [6].

resulting in analytic expressions for the various one-loop self-energies in terms of basic scalar master integrals. Next, we assemble the parameters of Eq. (1.1) and numerically evaluate the results using the program package `LoopTools` [24]. We observe that all ultraviolet singularities cancel between the different self-energies in the oblique parameters. On a more technical note, the matrix  $\gamma^5$  is treated naively (that is, anticommuting) with  $(\gamma^5)^2 = \mathbb{1}_{4 \times 4}$ , while we have checked explicit gauge parameter independence.



**Figure 2.** Left: Oblique parameters  $S$  and  $T$  in the NMSSM in the constrained case with central parameters  $\tan(\beta) = 1.7$ ,  $M_0 = M_{1/2} = 500$  GeV,  $A_0 = -800$  GeV,  $A_\kappa = -1500$  GeV,  $\text{sgn}(\mu) = +$ , and  $\lambda = 0.5$  (second row in Tab. 1) with variation of parameters as indicated in the figure. Right: Same as left but with central parameters  $\tan(\beta) = 5$ ,  $M_0 = 100$  GeV,  $M_{1/2} = 200$  GeV,  $A_0 = -700$  GeV,  $A_\kappa = -75$  GeV,  $\text{sgn}(\mu) = +$ , and  $\lambda = 0.2$ .

### 3 Results

As a simple numerical example we assume unification of all scalar masses  $M_0$ , fermion masses  $M_{1/2}$ , and trilinear couplings  $A_0$  (except for  $A_\kappa$ , which is considered separately) at the GUT scale. This scenario is usually called constrained NMSSM (cNMSSM). Furthermore, the ratio of the vacuum-expectation value of the two Higgs-boson doublets,  $\tan(\beta)$ , the Higgs coupling parameter  $\lambda$ , and the sign of  $\mu$  have to be fixed in addition to the parameters of the SM. The computation of the mass spectra and mixing angles at the electroweak scale is performed with the program package **NMSPEC** [17]. Let us note that our calculation of the oblique parameters is performed in the general NMSSM such that the oblique parameters for arbitrary parameter values can be easily computed. The program code for the oblique parameters is available as C-code from the URL [25].

The explicit values for the NMSSM parameters we choose in our numerical examples are given in Table 1. These parameter sets are inferred from the figures presented in Ref. [17]. The scales at which the NMSSM parameters are fixed are written as a superscript, with MSUSY and GUT the supersymmetry breaking scale, respectively, the grand unification scale – both scales are derived from the input parameters in **NMSPEC**.

In Figs. 1, 2 we present the results for the oblique parameters  $S$  and  $T$  for the different parameter sets given in Table 1. All other oblique parameters turn out to be rather small and are therefore not shown explicitly. In the Figures we vary successively the parameters  $A_0$ ,  $\tan(\beta)$ ,  $M_0$ , and  $M_{1/2}$  about the central values from Table 1 as indicated in the figures

(where we suppress the superscripts MSUSY and GUT). From the lines we see how the oblique parameters  $S$  and  $T$  change under variations of the parameter values. We also draw the  $1\sigma$  and  $2\sigma$  error ellipse corresponding to the recent experimental fits to  $S$  and  $T$  [6]:

$$S = 0.01 \pm 0.1, \quad T = 0.03 \pm 0.11, \quad \rho = 0.87. \quad (3.1)$$

Here,  $\rho$  denotes the correlation coefficient. Note that in this fit a SM Higgs-boson mass of  $m_{H_{\text{SM}}} = 117$  GeV is assumed, which we also use consistently as a parameter value in the SM self-energies.

As expected, in our numerical examples we find suppressed contributions to the oblique parameters  $V$ ,  $W$ ,  $X$ , which is due to the large masses of the additional particles as compared to the electroweak scale. From the sensitivity of the oblique parameters under variations of NMSSM parameters is clearly visible: for the central parameter set 1 in Table 1 we infer from Fig. 1, that the  $2\sigma$  error ellipse constrains  $\tan(\beta) \lesssim 40$ ,  $M_0 \gtrsim 250$  GeV, and  $M_{1/2} \lesssim 650$  GeV. For the other central values in Table 1 we can easily read off the constraints from Fig. 2.

## 4 Conclusions

For a large class of models beyond the Standard Model, the so-called oblique parameters give very sensitive constraints coming from electroweak precision measurements. We have computed the set of extended oblique parameters  $S$ ,  $T$ ,  $U$ ,  $V$ ,  $W$ ,  $X$  for the next-to-minimal supersymmetric model (NMSSM).

We have presented numerical examples with the parameters of the NMSSM chosen in a constrained case, as explained in Sec. 3. We observe the oblique parameters  $S$  and  $T$  to be highly sensitive on variations of the model parameters. In fact, fairly modest changes of the NMSSM parameters easily violate the constraints from the electroweak precision measurements.

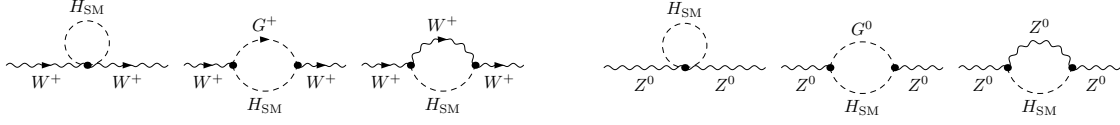
The oblique parameters have been computed for the general case, in particular with a general CP violating Higgs sector, such that they may be applied to arbitrary parameter values, in a more complete parameter scan, which we reserve for future work.

## Acknowledgments

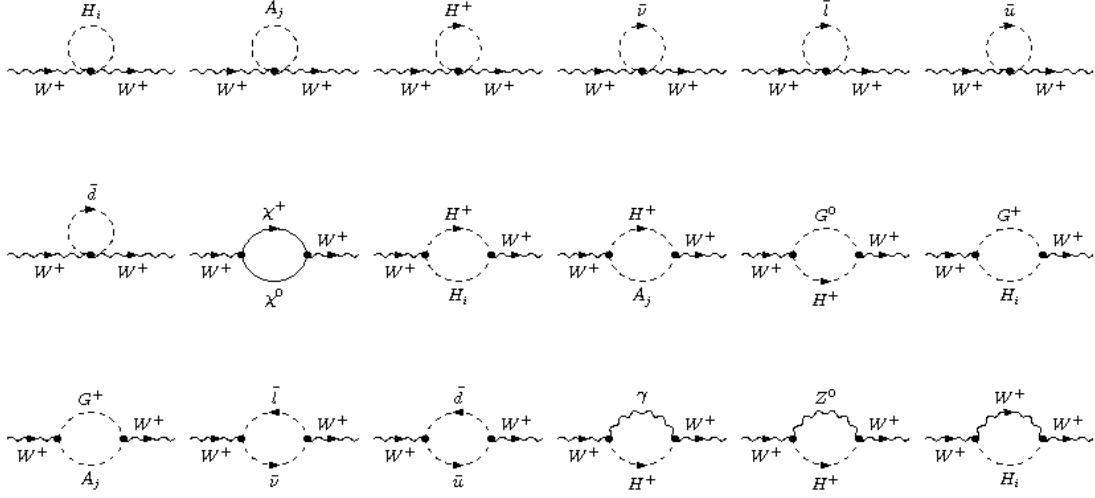
We are grateful to B. Fuks for quick response concerning an update of the NMSSM model file contained in `FeynRules`. The work of Y.S. is supported by the Heisenberg program of the Deutsche Forschungsgemeinschaft (DFG), contract no. SCHR 993/1.

## A Feynman diagrams for the oblique parameters

Here we present the Feynman diagrams which contribute to the oblique parameters of Eq. (1.1). For self-energy diagrams which exclusively have leptons, quarks, and gauge bosons in the



**Figure 3.** Feynman diagrams for the self-energies  $\Pi_{WW}^{\text{SM}}(s)$  and  $\Pi_{ZZ}^{\text{SM}}(s)$  which contribute to the oblique parameters. All other diagrams vanish, respectively, cancel with the corresponding diagrams in  $\Pi_{WW}^{\text{NMSSM}}(s)$  and  $\Pi_{ZZ}^{\text{NMSSM}}(s)$ .



**Figure 4.** Feynman diagram contribution to the self-energy  $\Pi_{WW}^{\text{NMSSM}}(s)$

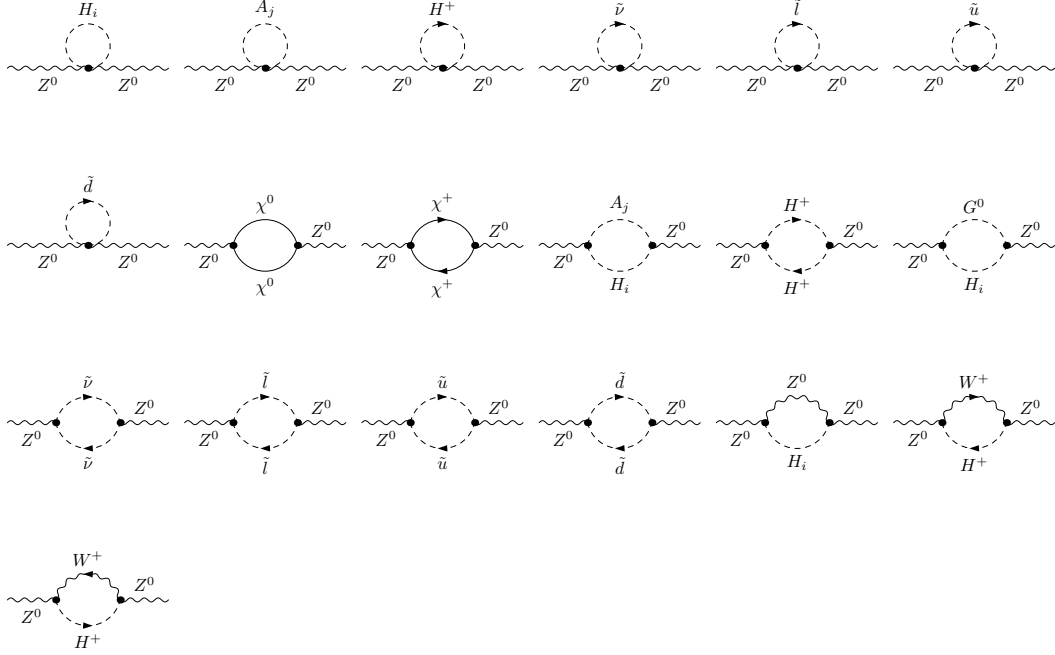
loops, the contributions to  $\Pi_{G_1 G_2}^{\text{NMSSM}}(s)$  and  $\Pi_{G_1 G_2}^{\text{SM}}(s)$  exactly cancel in Eq. (1.2) and do not have to be computed.

The contributions to  $\Pi_{G_1 G_2}^{\text{SM}}(s)$  consist of diagrams which contain the SM Higgs boson ( $H_{\text{SM}}$ ) in the loop. There are only contributions of this kind to the  $W^+$  and  $Z^0$  self-energies as shown in Fig. 3.

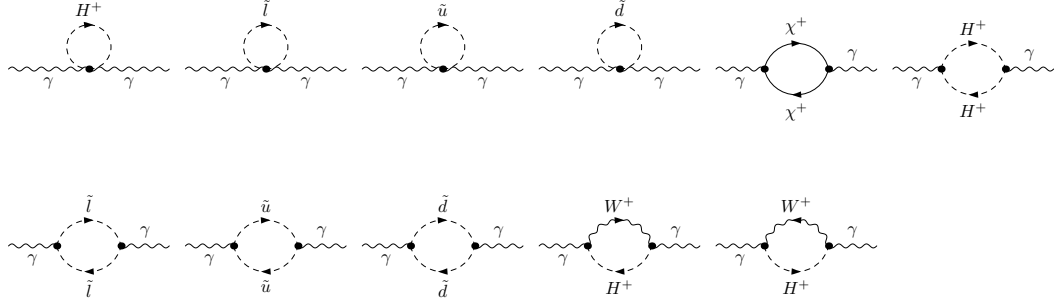
We also show all self-energy diagrams contributing to the NMSSM part of the oblique parameters. These diagrams involve scalar neutrinos  $\tilde{\nu}$ , scalar leptons  $\tilde{l}$ , scalar up- and down-type quarks  $\tilde{u}$ ,  $\tilde{d}$ , neutralinos  $\chi^0$ , charginos  $\chi^+$ , as well as the neutral Higgs bosons  $H_i$ ,  $A_j$ , the charged Higgs bosons  $H^\pm$  as well as the Goldstone bosons  $G^0$ ,  $G^+$ . All other contributions, for instance the self-energy with a lepton loop, cancel with the corresponding SM contribution.

The  $W^+$ ,  $Z^0$ , photon,  $Z^0$ -photon self-energy diagrams are shown in Figs. 4, 5, 6, 7, respectively.

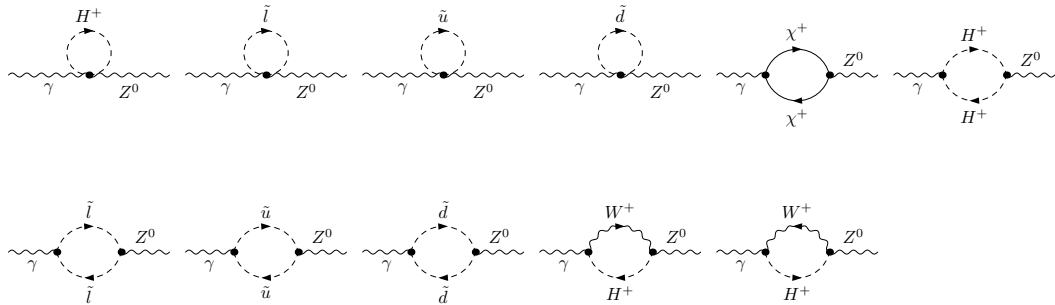




**Figure 5.** Feynman diagram contribution to the self-energy  $\Pi_{ZZ}^{\text{NMSSM}}(s)$



**Figure 6.** Feynman diagram contribution to the self-energy  $\Pi_{\gamma\gamma}^{\text{NMSSM}}(s)$



**Figure 7.** Feynman diagram contribution to the self-energy  $\Pi_{Z\gamma}^{\text{NMSSM}}(s)$

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url [www.physik.uni-bielefeld.de/~maniatis/NMSSM/oblique](http://www.physik.uni-bielefeld.de/~maniatis/NMSSM/oblique).